Shape of the unbent guitar sides

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1) Overview

Considering the fact that the back of a guitar has a curved surface from the tail to the neck and from the center line in the back towards the waist, upper and lower bouts, the sides must follow a certain line to accomodate to this curved surface in the back. A typical shape of the sides can be seen in the next figure:

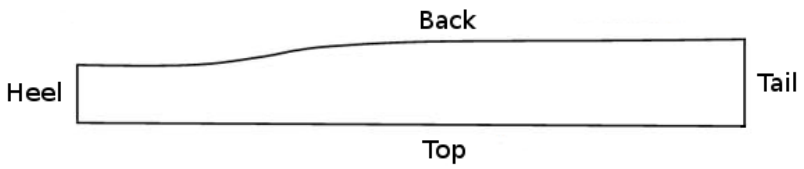


Figure 1 – Typical shape of an unbent guitar side

In order to obtain the shape mentioned in Figure 1, we have to solve a 3-dimensional problem and set the surface of a sphere with a certain radius (around 15 feet) that must touch the tip of the heel and also the tip of the tail (points A and B of Figure 2). Also, all the points in the edge of the back should belong the the same surface. To help visualize the case, Figure 2 shows three orthogonal planes with the guitar contour placed on them.

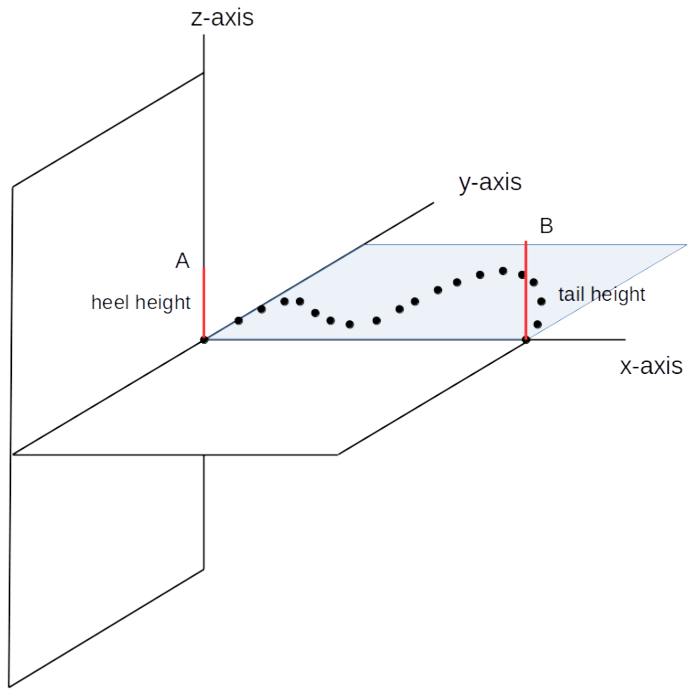


Figure 2 – Three orthogonal planes with the guitar shape placed on the X-Y plane

If we look perpendicular to the X-Z plane, we can see something like that:

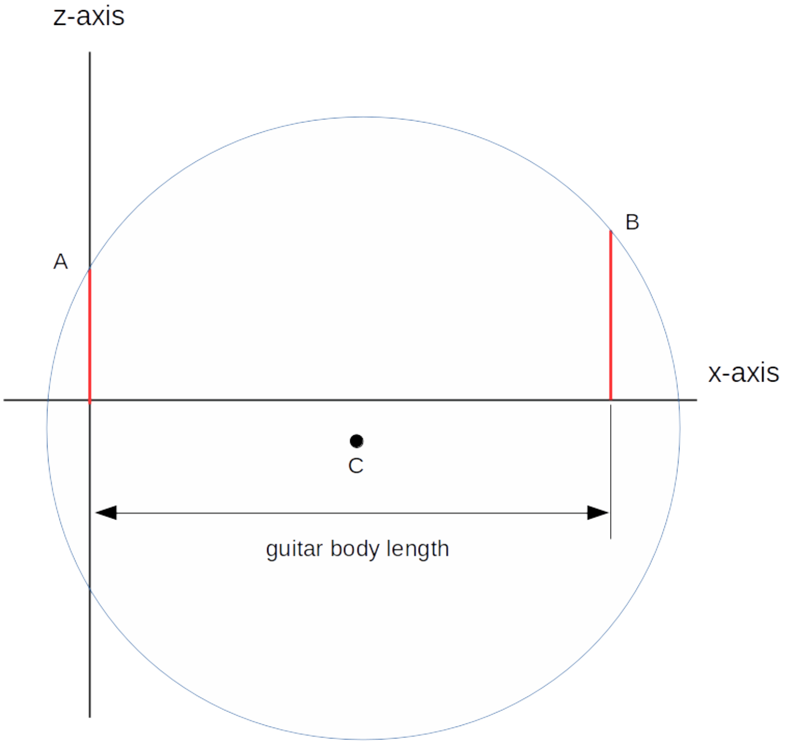


Figure 3 – Vision from the X-Z plane (A and B are the heights of the heel and tail, respectively)

Please note that when this is done, the sphere transforms into a circle, where C is the center of that circle, and most important, **point C belongs the X-Z plane, where y = 0 !!!** (refer to Figure 2 to see the fact)

In order to give a perfect definition of the problem, some values where considered to show the design and with few changes on those parameters, anyone can fit this design to his/her particular needs.

In the present case the parameters and dimensions necessary to the design where:

a) guitar contour (my case is shown in Figure 4)
b) guitar body length - 482 mm
c) heel height – 88 mm
d) tail height – 98 mm
e) radius of the guitar back – 15 feet (15 x 12 inches x 25,4 mm) = 4572 mm

It is good to mention that a radius of 4572 mm is 4,57 meters !!! It is a huge sphere of 9,1 meters of diameter !!! Figure 3 is very out of scale but is good enough to give an idea of the situation.

2) Mathematics behind the facts !!!

Step a) Guitar contour

Before we get to some math, in order to complete the design, it is necessary to define the contour of your guitar (step a, from the previous section). I drew the contour in a paper and placed points in the X-axis spaced of 5 mm each. I got across the 482 mm (my body length) 104 points. Figure 4 shows my job. Near x = 0, I put 4 more points to have a better definition of the curve. Near x = 482 mm, I add 2 more points for the same reason.

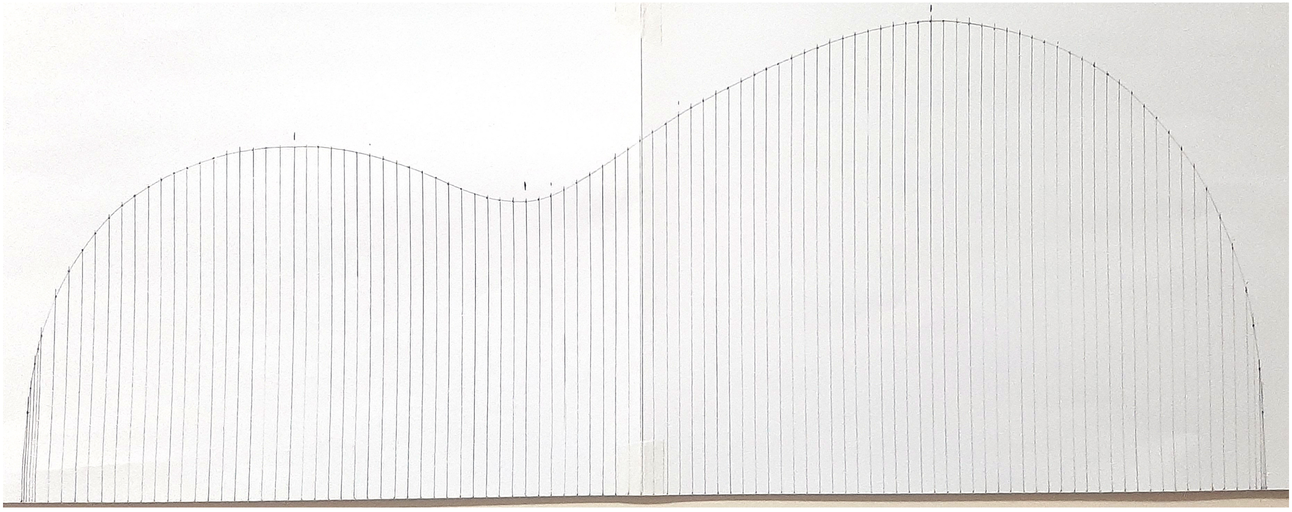


Figure 4 – Guitar contour

At this point I have a collection of points. The x-axis varies from 0 to 482 mm and the y-axis varies from 0 to 185.2 mm (lower bout). It can be considered other methods to obtain this important step of the design.

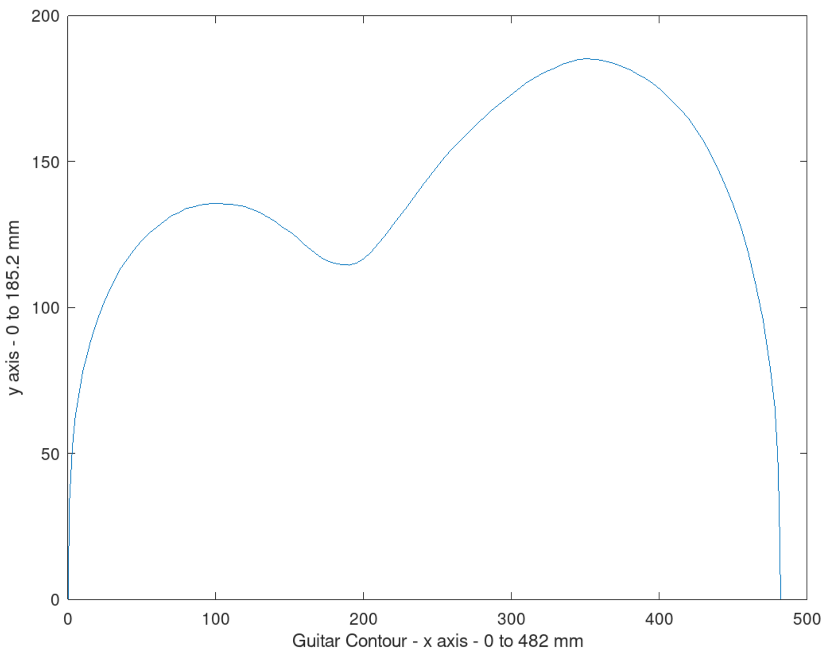


Figure 5 – Guitar contour from Figure 4 plotted on Octave™

Steps b, c, d and e)

These steps are straightforward. Is only to measure and define the guitar body length (482 mm), the heel height (88 mm), the tail height (98 mm) and the radius of the guitar back (15 feet – 4572 mm).

3) Determination of the Sphere Center

As mentioned before, when we have a look perpendicular to the X-Z plane, the problem transform itself into a solution of a 2-dimension case. So, we have in fact to determine the center of a circle.

The equation of a sphere is given by:

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2 \qquad (Eq.1)$$

where a, b and c are the coordinates of the center in the planes X, Y and Z, respectively
r is the radius of the sphere.

By the other hand, the equation of the circle in the X-Z plane is given by:

$$(x-a)^2+(z-c)^2=r^2 \qquad (Eq.2)$$

Considering that:

for x = 0 ; z = hheel = 88 mm and
for x = blen = 482 mm ; z = htail = 98 mm

If we substitute those values on Eq. 2, we have:

$$\begin{aligned} (-a)^2+(88-c)^2 &= 4572^2 \qquad (Eq.3) \\ (482-a)^2+(98-c)^2 &= 4572^2 \end{aligned}$$

The (Eq.3) it is a system of equations that can be solved by numerical computation. I am using Octave™ for doing it. For this task , also can be used Scilab™ and Matlab™. There are minor differences between them.

The Octave™ function for solving (Eq.3) is ***fsolve***. Next is seen how to create the function and how to use it.

Once you are inside Octave™ is possible to use its editor or use another general text editor and type the next lines:

```
function y=c_center(x)
global hheel htail blen radius
  y=zeros(2,1);
  y(1)=(-x(1))^2+(hheel-x(2))^2-radius^2;
  y(2)=(blen-x(1))^2+(htail-x(2))^2-radius^2;
endfunction
```

It can be seen that y(1) and y(2) are the equations described by (Eq.3). All those 6 lines must be typed and saved in a text file named as *c_center.m*. This file (c_center.m) must be seen as a subroutine of a main file and has the purpose to calculate de center of a sphere, the parameters a and c. So, in the main file, that will be called here *guitarsides.m*, there will be a way to activate the c_center function. The next line is the one that must be placed inside the *guitarsides.m* in order to have the center determined:

[k,fval]=fsolve('c_center',[0;-radius]);

where k is a vector containing the solution for (Eq.3), the values a and c
fval is an error evaluation for the solution (should be very small for a good solution)
'c_center' is the name of the function subroutine that computes the sphere center and,
[0;-radius] is the adopted initial condition for start searching the solution.

Observation must be done here about the initial condition [0;-radius]. This set of conditions will lead to a solution that will produce a spherical *convex* surface over the guitar back. Which is considered the "normal" approach.

If we set the initial conditions to [0;+radius], the solution will be the other possible solution for (Eq.3), that will lead to a spherical *concave* surface for the guitar back, that is a bit unusual. But for someone who wants to try this awkward guitar back, feel free to do that. A small change in a equation signal must be done besides the initial condition also has to be changed to [0;+radius]. This will be said a little bit further.

For the data considered so far, after executing the *c_center* routine, the encountered value for k will be:

k =
335.70
-4471.66

Note that a = 335.70 mm and c = -4471.66 mm

If the choice for initial conditions where [0;+radius], the result would be (the concave solution):

k =
146.30
4657.66

4) Determination of Z coordinates

Now that we have the x-file , the y-file (x-file and y-file describe the guitar contour) and the sphere center (a and c), we have conditions to determine the amplitudes of the guitar sides (amplitudes on the z-axis) for each pair (x,y) that are in the x and y-files. For doing this it is only necessary to evaluate (Eq.1), repeated here for convenience:

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2 \qquad (Eq.1)$$

We need to remember now that parameter b is equal to 0, once the sphere center lies on the X-Z plane.

After a small manipulation of (Eq.1), we can get z as a function of x and y:

$$z = c + \sqrt{(r^2 - (x-a)^2 - y^2)} \qquad (Eq.4)$$

Equation (Eq.4) is a collection of points that perfectly match the guitar back border to the sides of the guitar.

Now that (Eq.4) is derived, for those who want to make a concave guitar back design, (Eq.5) must be used instead:

$$z = c - \sqrt{(r^2 - (x-a)^2 - y^2)} \qquad (Eq.5)$$

Remember to set initial conditions to [0;+radius], otherwise the sphere center will be in a wrong place.

5) Plotting results and getting a file template

Before plotting results, we need to remember that the points in the vector X describe their position related to the guitar body length (heel to tail) and not to the entire contour length.
To handle this fact, a simple drawing can be used to illustrate:

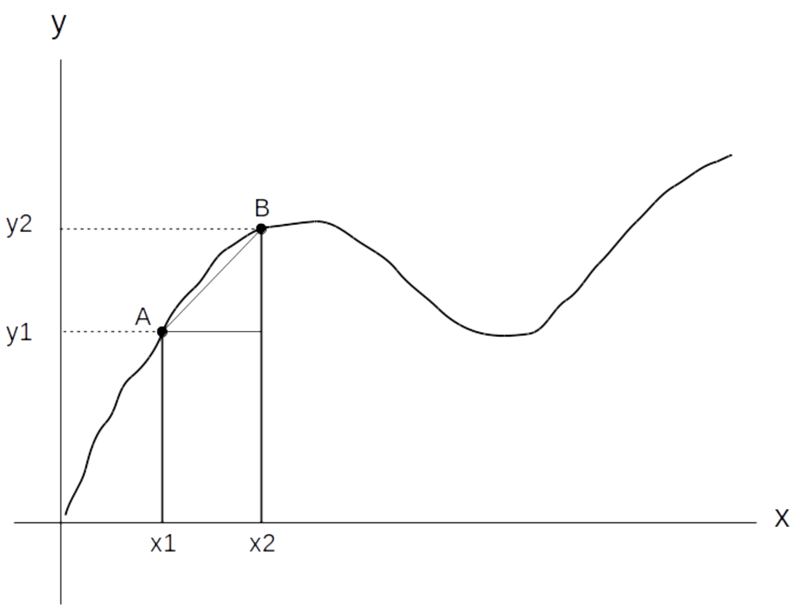


Figure 6 – Points in the vectors X and Y

If we desire to plot the real size of the guitar sides we need to add up all distances AB shown in the example Figure 6.

To calculate AB distances, once they are the hypotenuses of triangles, they are given by:

$$AB = \sqrt{(x2 - x1)^2 + (y2 - y1)^2} \qquad (Eq.6)$$

This task can be easily implemented in Octave™ by using the next commands:

distance=sqrt(diff(x).^2+diff(y).^2);
comp=cumsum(distance);
lin=[0 comp];

Once vectors x and y are 104 points long, after completing the first line above, the vector called *distance* will contain a collection of all calculated distances between consecutive points. Notice that every two points lead to just one distance. By doing that, vector *distance* will be only 103 points long. The second line is necessary to implement a summation of all consecutives distances AB, forming an increasing vector containing all the distances AB. The third line only includes a 0 value in the beginning of the points to have again 104 points.

The method increases precision as the amount of points in the vectors x and y also increases.

6) Listing of files *guitarsides.m* and *c_center.m*

guitarsides.m ✖ | c_center.m ✖

```
global hheel htail blen radius
%
% hheel - Height of the heel
% htail - Height of the tail
% blen - body length
% radius - radius of the back curvature

%           ****** USER SECTION ******
%
hheel=88;htail=98;blen=482;radius=15*12*25.4;
x=[0 1 2 3 4 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140 1
y=[0 34 43 52.5 58 63 78 88 96 102.5 108 113 116.5 120 123 125.5 127.5 129.5 131.5 132.5 134 134.5 135.2 1
%
%       ****** END OF USER SECTION ******
%
plot(x,y)
xlabel('Guitar Contour - x axis - 0 to 482 mm')
ylabel('y axis - 0 to 185.2 mm')
distance=sqrt(diff(x).^2+diff(y).^2);
comp=cumsum(distance);
lin=[0 comp];
figure,plot(x,lin)
xlabel('Guitar contour - Total side length ( 720.49 mm ) as a function of the body length (482 mm)')
[k,fval]=fsolve('c_center',[0;-radius]);
a=k(1);
c=k(2);
z=c+sqrt(radius^2-(x-a).^2-y.^2);
figure,plot(lin,z)
xlabel('Total side length - 0 to 720.486 mm')
ylabel ('Side height - 88 to 98 mm')
axis([0 725 0 725])
A=[lin; z];
id=fopen('template.txt','w');
fprintf(id,"%4.3f   %4.3f \n",A)
fclose('template.txt');


```

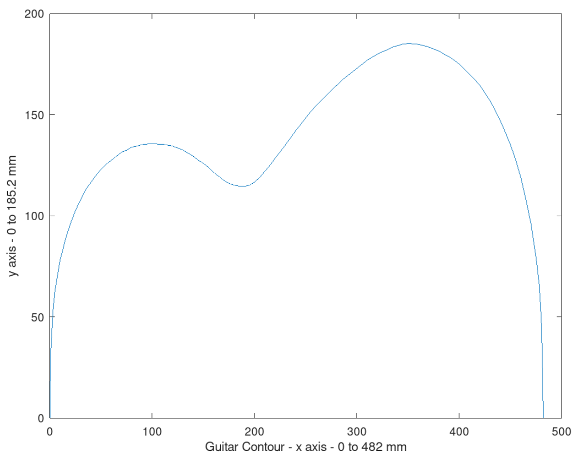
Figure 7 – File *guitarsides.m*

The last three lines in the file above, show that at the end of execution, a file called *template.txt* will be generated. That file can be used to make a template in paper or, using a CNC machine, produce a template in wood, plastic or metal sheet.

```
function y=c_center(x)
global hheel htail blen radius
  y=zeros(2,1);
  y(1)=(-x(1))^2+(hheel-x(2))^2-radius^2;
  y(2)=(blen-x(1))^2+(htail-x(2))^2-radius^2;
endfunction
```

Figure 8 – File *c_center.m*

7) Plotting results



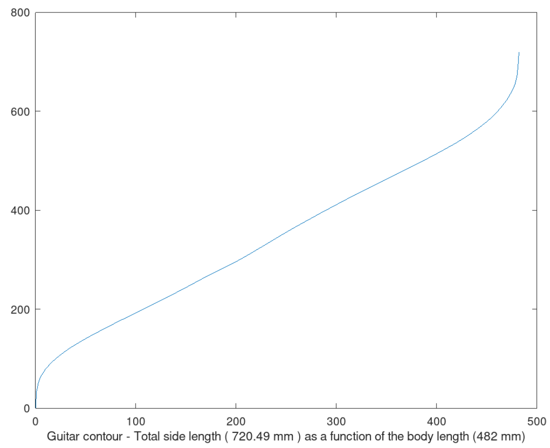


Figure 9 – a) Guitar contour b) Total side length

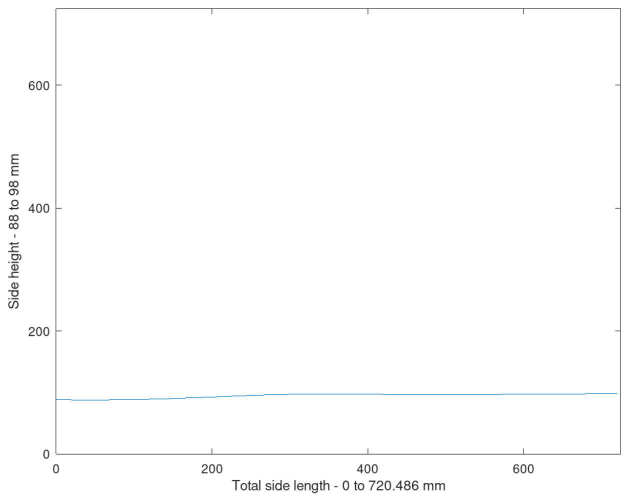


Figure 10 – Shape of the designed guitar sides

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